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EDUCATIONAL SIMULATION OF BIOELECTROMAGNETISM FEATURES OF LIVING ORGANISMS

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Abstract: *The paper deals with a topic of computer simulation possibilities of bioelectromagnetical features and usage of this simulations for demonstration purposes of educational process in conditions of interdisciplinary area of pedagogical process oriented as well towards to technical students as to students of medicine. Such new approach has been established in the Department of theory of electrical engineering of the University of West Bohemia in Plzeň under leading of Mr.Prof.I.Doležel.*

Key words: *Bioelectromagnetism, living organisms, computer simulation, interdisciplinary research and education, MATLAB*

INTRODUCTION

Today's research is crossing over a borders of individual scientific areas. If modern education has to follow this rapid development speed it is necessary to trace the same path. This paper is focused on interdisciplinary education of selected bioelectromagnetic features of cardiac cells – namely ion currents through cell's membrane.

This topic is a part of mosaic of disciplines studied and taught in the new study area established by Mr. Prof. Ing. I. Doležel, CSc. who is the executive chief of the department of theory of electrical engineering in the Univeristy of West Bohemia in Plzeň. This education is dedicated as well for medicine student as the students of electrical engineering.

So it must cover mathematical basics, electric properties of cell's membrane and ion current kinetics in comprehensive form which is acceptable for both mentioned groups of students. Also computer simulation techniques are necessary to demonstrate mathematical and programming attitude as well as biological substance. It is necessary to explain an approach enabling dealing of such multidisciplinary education. It has two basic levels which are mathematical and biological one and both of these levels must mutually correspond.

Hence the goal of this paper is neither pure mathematical theory or computer modelling nor biological study but joining of all these disciplines into common approach leading towards to new horizons of biomedical and engineering possibilities which could be used to decrease the pain of human beings and the Nature in general.

Subsidiary but not less important goal is to contribute to widening of study range of the all students of this new study area.

1 MATHEMATICAL APPROACH

Mathematical approach must respect bioelectrical substance of researched phenomena and must be acceptable for students of medical departments. Also this mathematical approach must be rich enough for technically oriented students to develop own new models and simulation programs yielding new results for biomedical practice. A level of an accuracy of experimentally measured bioelectromagnetic data is determined mainly by the time of sampling frequency and signal accuracy of the data acquisition technology which is usually D/A measuring PC card. It causes that the resultant curve of biological object behaviour looks like a non-analytical function. The Hausdorff space is suitable for description this situation because its main feature is that any two different points can be distinguished.

In this space there is necessary to define any appropriate set of functions which are universal enough to express measured medical data and on the other hand these functions must not be many. This task is similar to well known task of finding minimal set of functions for binary logical circuits realisation. Another necessary criterion is that these functions should have appropriate number of parameters which could be identified with a parameters of external factors having significant influence on the measured biological objects. These factors are e.g. level of exciting voltage applied on a cell, drugs, etc.

Keeping in mind all of this one could select a set of functions based on the exponential curve. These functions were also used in [4] however this paper brings new way of combinations of them. Classical mathematical techniques are used also of course, see e.g.[2], [3].

Another feature of new approach is utilisation of space transformations changing its dimension. By this way it is possible to create a projection of a smooth function into a curve which is not smooth. This part of education is ideal for simulation either by using computer or personally because this moment is new and never mentioned in available mathematical literature. This is illustrated in the following figures – see following text.

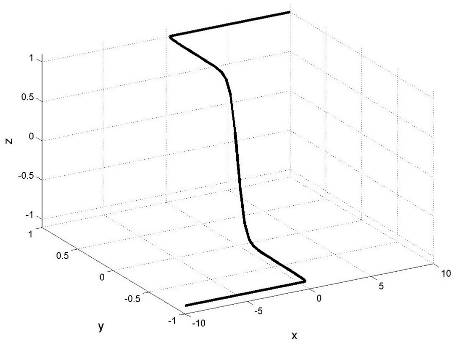

Fig.1: 3D into 2D function matting

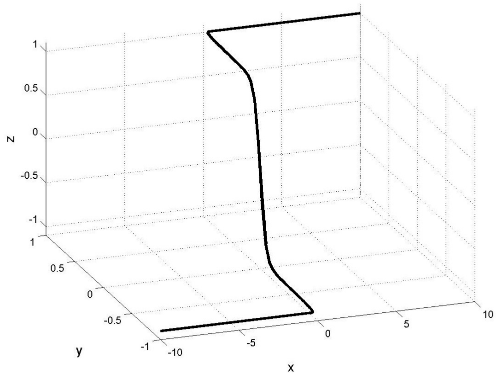

Fig.2 : 3D into 2D function matting

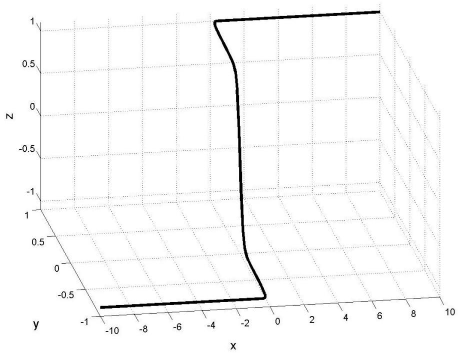

Fig.3 : 3D into 2D function matting

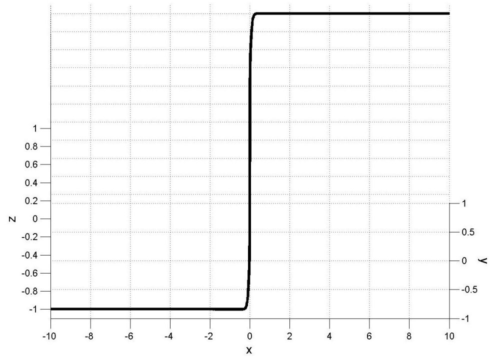

Fig.4 : 3D into 2D function matting

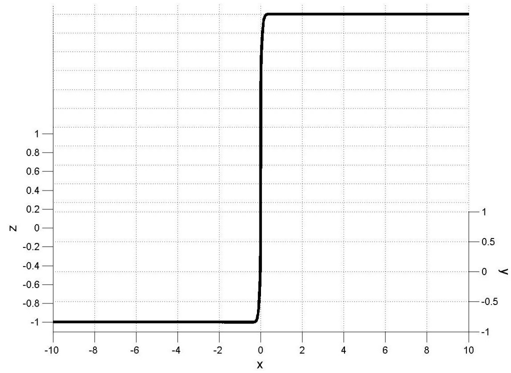

Fig.5 : 3D into 2D function matting

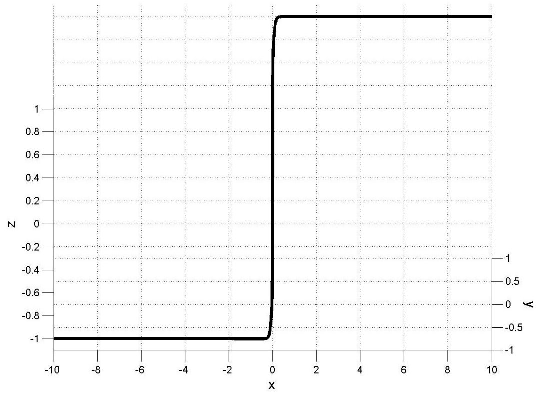

Fig.6 : 3D into 2D function matting

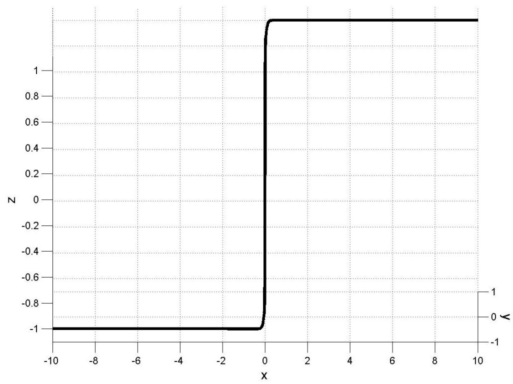

Fig.7 : 3D into 2D function matting

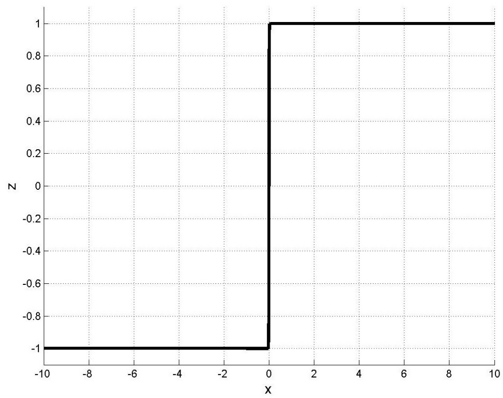

Fig.8 : 3D into 2D function matting

The text for previous figures 1 to 8 is the same because the function is the same.

Fig.1 presents 3D smooth function described by these formulae :

$$\mathbf{y} = \tanh\left(k \cdot \mathbf{x}\right) \tag{1}$$

$$\mathbf{z} = \tanh\left(k \cdot \mathbf{y}\right) \tag{2}$$

where $\mathbf{x} \in \left\langle -10, +10 \right\rangle$

and $k = 10$ is a coefficient having meaning of slope factor of this function. This is an empirical value of this coefficient and was set experimentally as well as the function itself. In Fig.1 to Fig.4 there is the function (2) shown in different azimuth and in Fig. 5 to Fig.8 there is the same function in different elevation above x-y plane.

The resulting "step like" function is in Fig.8. This is a typical, although not mentioned in mathematical literature, example of practical usage of external law of composition in form of the mapping changing number of dimensions.

This example is available in demo_3D.zip file enclosed to this paper on CD.

2 BIOLOGICAL BACKGROUND

Usage of mathematical techniques must correspond to studied biomedical objects. They are described in detail e.g. in [1], [4], [5], [6]. These literature sources contain very often biomedical data obtained in laboratory experimental condition and hence such data are loaded by errors caused by technology limitations. Especially data acquisition PC cards are sources of such errors which are difficult to avoid. These cards have limited number of bits of digitalized analogue to signal conversion and are also limited by sampling frequency in accordance with Shannon sampling theorem. These facts are to be kept in mind in dealing with such data. It was partially mentioned at beginning paragraph dedicated to mathematical approach and space kind choice.

Also sometimes the measured data are presented in form of silhouette shapes and hence technicians must use approximate approach which is as much as possible exact to express obtained data character.

This was a goal of many researcher and scientists however they did use traditional mathematical approach which is based on basic mathematical functions described e.g. in [3]. But these functions can not follow the real functional dynamics of bioelectric activity of cardiac cells. The cause was partially explained in the example of the mapping changing number of dimensions above.

An example of real dynamics shapes of functional curves is in Fig.9.

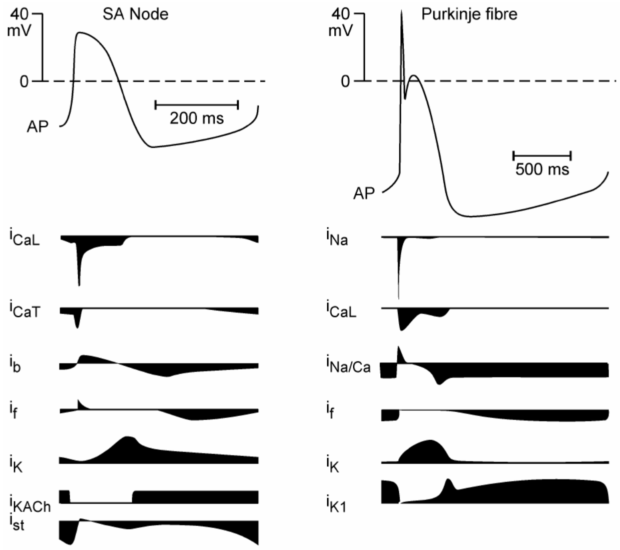

Fig.9 : Example of ion currents

In Fig.9 there are an examples of really measured ion currents in cell's membrane of cardiac cell. There are some facts to be noticed :

These shapes are sometimes printed without exact axes and units (voltage, time). The shapes are rapidly varying in dependence on a place of the heart locality where they are measured and these differences are not only in amplitude but also in the time, frequency resp., of signals length. These shapes are only an informative ones because they can be very significantly modified by external influence factors like as physical tiredness, drug effect, etc.

All these facts must be taken into consideration in a conception of the new mathematical model because this model must contain "appropriate" number of parameters which can be identified with these external parameters and by this way they can serve for prediction of the effect of external factor on cardiac muscle.

3 FUNDAMENTAL FUNCTIONS SET

All the facts mentioned above seem to be difficult to deal with via classical mathematical approach. New approach presented herein offers however absolutely another way of solution of defined problem.

Beside using Hasdorff space and the mapping changing space dimensions of observed functions one can define a set of fundamental functions which are suitable for almost exact approximation of the biomedical data looking like ion currents e.g. in Fig.9. This step can be compared with a choice of minimal functional set of logical functions which are being used for logical circuit realization. Here the approach looks to be more complicated because in this case there is immense number of real and complex functions and their parameters and also this problem was not dealt with in any mathematical literature. However it can be solved. Some functions look to be very promising like e.g. spline function but such approach leads to a solution with too much parameters which are hardly to be identified with limited number of external influence factors acting on cardiac cells.

Here we will use the similarity aspect and define functions which can serve, in mutual combination, as building stones yielding almost any shape of the ion current being measured on cardiac cell's membrane. This fundamental functions set contains these functions :

- rounded step function
- slope hill function
- symmetric hill function
- slide function
- wave function

Now we will define and illustrate these functions without detail mathematical derivation which is not important for education from biomedical point of view.

3.1 Rounded step function

This function can be identified very often in ion currents dynamics curves and it is caused by cardiac cells ion membrane channels properties. Definition of this function is this :

$$y = am_0 \qquad \text{for} \quad t < t_0 , \tag{3}$$

$$y = am_1 \qquad \text{for} \quad t > t_1 , \tag{4}$$

and for $t \in \langle t_0, t_1 \rangle$:

$$x = a \cdot \left(\frac{t - t_0}{t_1 - t_0} - 1 \right), \tag{5}$$

$$y_0 = \sqrt{\frac{r^2 - b^2 x^2}{a^2}} , \tag{6}$$

$$y = am_0 + y_0 \cdot \left(\frac{am_1 - am_0}{b} \right), \tag{7}$$

For the following parameters values this function has the shape depicted in Fig.10.

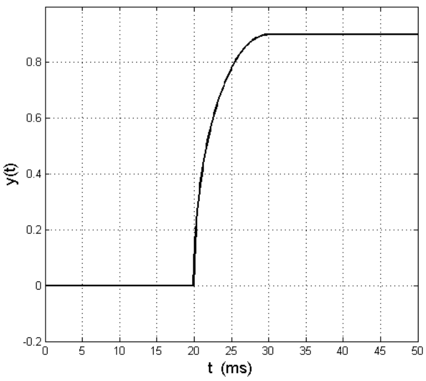

Fig.10 : Rounded step function example

This graph in Fig.10 was generated for these parameters : $t_0 = 20$ ms, $t_1 = 30$ ms, $am_0 = 0$, $am_1 = 0.9$, orientation "right".

3.2 Slope hill function

This function is defined by this way :

$$y = am \cdot \exp\left[-\left(\ln\left(t + t_a \right) \right)^2 \right] \tag{8}$$

for time point t from time vector $\boldsymbol{t}$ from the symmetric interval $t \in \langle -t_a, +t_a \rangle$, where am is the amplitude of the slope hill. Meaning of this function can be explained as model of time deformation caused by saturation of cell's interior by ion currents.

The illustrative graph of the shape of this function is in Fig.11 for these parameters values : $-t_a = 0$, $+t_a = 500$ and $am = 2.5$.

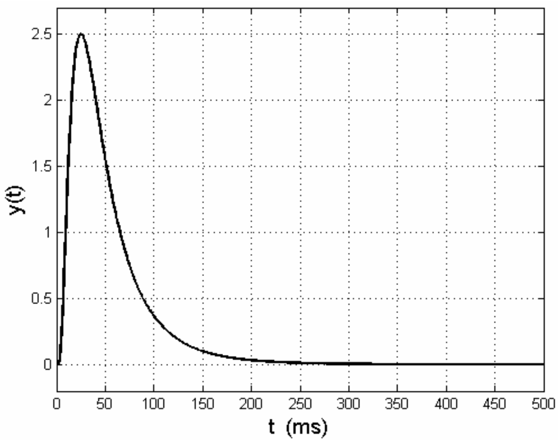

Fig.11 : Slope hill function example

By changing the sign of amplitude am and by using MATLAB parameter 'orientation' can be changed declination and polarity of this hill shape.

3.3 Symmetric hill function

This kind of function also very often occurs identify in ion currents experimentally measured shapes. It can be defined by this way :

$$\mathbf{y} = 1 + am \cdot \exp\left[-\left(s \cdot \left(\mathbf{t} - t_0 \right) \right)^2 \right] \tag{9}$$

where : $\mathbf{t}$ is vector of time points, s is slope factor of the function, am is the amplitude of the hill decreased by 1 and t_0 is the centre of the hill which means position of its top in time. Illustrative example is in Fig.12 where these concrete parameters were used : $t \in \langle 0, 500 \rangle$, $t_0 = 250$, $s = 0.05$, $am = 0.7$.

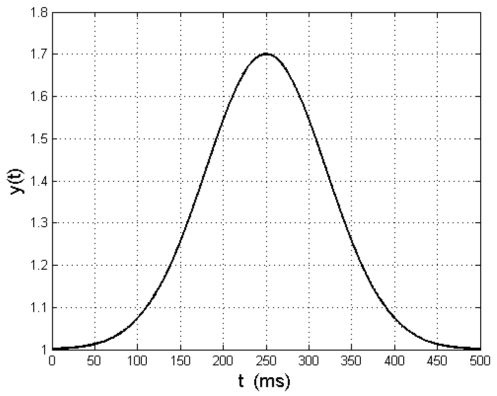

Fig.12 : Symmetric hill function example

3.4 *Slide function*

This function simulates strong start of ion current followed by slow exponential decrease. It is another of dynamics of basic ion current types. This function can be defined by this way :

$$y = 0 \qquad \text{for} \quad t < t_0 , \tag{10}$$

$$y = am \qquad \text{for} \quad t = t_0 , \tag{11}$$

$$y = am \cdot \exp\left(-\left(t - t_0\right) \cdot s\right) \quad \text{for} \quad t > t_0 , \tag{12}$$

where t_0 is the starting time of the function, am is the amplitude, and s is the slope factor. An illustrative example is depicted in Fig.13 for these concrete parameters : $t_0 = 200$, $am = -2$, $s = 0.02$.

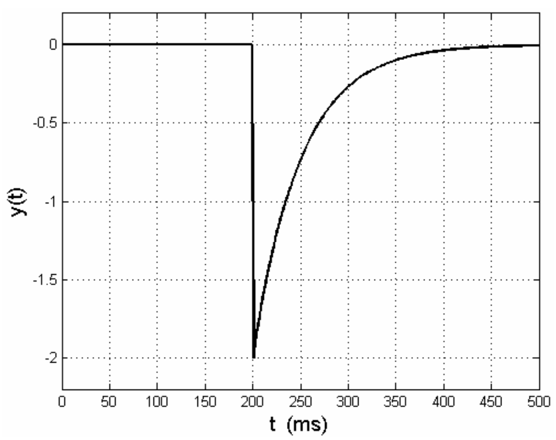

Fig.13 : Slide function example

3.5 *Wave function*

This is the last of fundamental functions used for ion current modelling. It is representing smooth start of ion current followed by smooth decrease with defined time delay between these two phases. The definition of this function is the following one :

$$n = \tanh\left(s_0\left(t - t_0\right)\right) , \ \text{for} \ n < 0 : n = 0 , \tag{13}$$

$$d = 0.5\left(1 + \tanh\left(s_1\left(t - t_1\right)\right)\right) , \ \text{for} \ d < 0 : d = 0 , \tag{14}$$

$$y = am \cdot \left(n - d\right) \tag{15}$$

where this function is sum of two $\tanh(*)$ -like functions. This functions enables to express also “deformed” waves which are typical for biomedical measurements. Function component denoted as n means starting part of the function, part d means ending part of the function and the resulting function y is the simple sum of them amplified by factor am . Illustrative example of this function is in Fig.14 for these parameters : $t_0 = 100$, $s_0 = 0.02$, $t_1 = 300$, $s_1 = 0.1$, $am = 1.5$.

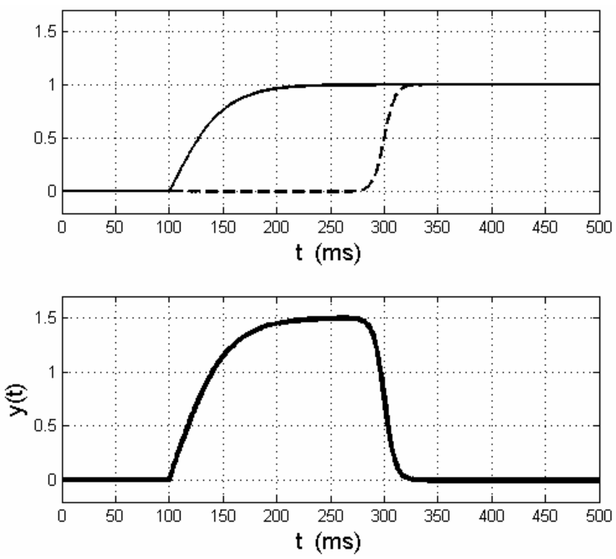

Fig.14 : Wave function example

4 CONCLUSION

The paper summarises principles of the new approach to exact modelling of biomedical laboratory obtained data, concretely ion current in cardiac cell’s membrane.

Presented mathematical approach together with fundamental functions set serves as solid demonstration base as well as for new educational area established by Mr.Prof.I.Doležel as the basic of new prediction methothodology useful in biomedical practice.

5 ACKNOWLEDGEMENT

Here is the place to give thanks to Mr.Prof. I. Doležel for his organizational leadership of the new research area and also for his valuable recommendations and to Mr.Assoc.Prof.M.Štengl for medical consultations.

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